Constraints on the Strength of Primordial Magnetic Fields from Big Bang Nucleosynthesis Revisited

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ABSTRACT

In this paper, we revisit in detail the effects of primordial magnetic fields on big bang nucleosynthesis (BBN) including a discussion of the magnetic field geometry and the anomalous magnetic moment. The presence of magnetic fields affects BBN by (1) increasing the weak reaction rates; (2) increasing the electron density due to changes to the electron phase space; and (3) by increasing the expansion rate of the universe, due both to the magnetic field energy density and to the modified electron energy density. Of the effects considered, the increase in the expansion rate due to the magnetic field energy is the most significant for the interests of BBN. The allowed magnetic field intensity at the end of nucleosynthesis (0.01 MeV) is about 2×10^9G and corresponds to an upper limit on the magnetic field energy density of about 28% of the neutrino energy density ($\rho_B \leq 0.28\rho_\nu$).

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1. Introduction

Big bang nucleosynthesis (BBN) provides an unique quantitative window for processes occurring in the early universe[1] between temperatures of 1MeV and 0.01MeV. The agreement between the light element abundances predicted by BBN and observations strongly constrains the dynamics of the universe at this epoch including the presence of strong magnetic fields. A primeval magnetic field existing during nucleosynthesis would have three major effects on BBN: (i) it would alter the weak interaction rates, (ii) it would modify the electron densities in phase space, and (iii) it would increase the cosmological expansion rate. Some of these effects were examined by a number of authors[2] and most recently by Cheng, Schramm, and Truran[3], Grasso and Rubinstein[4], and Kernan et al[5]. In this

paper, we revisit our earlier analysis[3] and find reasonable agreement between subsequent work by different authors[4,5] and our present results. Although some slight differences remain, the basic conclusions seem unambiguous. Here we also show that the effects of the spatial distribution of the magnetic fields and the anomalous magnetic moment do not affect significantly the results.

2. Three major effects of B fields on BBN

Jedamzik et al[6] have shown that neutrino decoupling effectively damps all magnetohydrodynamic modes up to the scales around a tenth of the Hubble radius at neutrino decoupling. If l_d is the largest scale over which the magnetic field becomes spatially homogeneous due to neutrino damping, then $l_d \approx 0.1 H^{-1}$ at $T \simeq 1$ MeV. This implies that if there are magnetic fields present during BBN their spatial distribution is very smooth on scales smaller than l_d and the field can be taken as constant within these scales.

The magnetic field spatial distribution needs to be taken into account if l_d is smaller than the length scales over which reactions and mixing occur during BBN. The relevant scale to be compared to l_d here is the largest mixing length which corresponds to the neutron diffusion length, d_n. Jedamzik and Fuller[7] showed that $d_n(1 \text{ MeV}) \lesssim 1$ m while the horizon $H^{-1}(1 \text{ MeV}) \simeq 10^8$ m. Since $l_d \gg d_n$, the magnetic field is constant within correlated volumes and will be taken as constant below.[1] We also assume that the field is randomly oriented within each volume of radius l_B such that the expansion rate is not anisotropic and that Robertson-Walker metric is valid.

In an uniform magnetic field with magnitude B chosen to lie along a z-axis, the dispersion relation for an electron propagating through the field is

$$E = [p_z^2 + m_e^2 + 2eBn_s]^{\frac{1}{2}} + m_e \kappa, \tag{2.1}$$

where $n_s = n + \frac{1}{2} - s_z,\ (n_s = 0, 1, ...)$, n is the principal quantum number of the Landau level and $s_z = \pm 1/2$ are spins. e is the electron charge, p_z the electron momentum, m_e the rest mass of the electron, and κ is the anomalous magnetic moment term[9] for an electron in the ground state ($n = 0$, $s_z = 1/2$). For relatively weak fields (i. e., $B \lesssim 7.575 \times 10^{16}$

[1] Recently, Grasso and Rubinstein[8] assumed that the magnetic field during BBN had fluctuations on the scale of the horizon at the electroweak transition which is of the order of the diffusion length at the end of BBN. If damping due to neutrino decoupling were not as effective as found in Ref.[7], the spatial variations of the magnetic field could affect BBN outcome.

G), $\kappa = -\frac{\alpha_e}{4\pi}\frac{eB}{m_e^2}$, while for stronger fields, $\kappa = \frac{\alpha_e}{2\pi}(\ln\frac{2eB}{m_e^2})^2$, where $\alpha_e = \frac{1}{137}$. The number density of states in the interval dp_z for any given value of n_s in the presence of magnetic field is described by[10]

$$(2-\delta_{n_s0})\frac{eB}{(2\pi)^2}dp_z. \tag{2.2}$$

We now discuss how Eqs.(2.1) and (2.2) affect BBN in detail.

2.1 Weak reaction rates:

The weak interaction rates in a constant magnetic field without QED correction have been derived by Cheng, Schramm, and Truran[11]. In an expanding universe, the B field evolves as R^{-2}, where R is the scale factor of the universe. During BBN, $R \propto T_\nu^{-1}$, where T_ν is the neutrino temperature. Let B_i represent the magnetic field at an initial temperature $T_i = 1$ MeV, $\gamma_i = B_i/B_c$ and $\gamma = B/B_c$, where $B_c = m_e^2/e = 4.4\times 10^{13}$G is the critical field at which quantized cyclotron states begin to exist. The magnetic field at any temperature T can then be written as

$$B = B_c\gamma_i(\frac{T_\nu}{T_i})^2 \quad \text{or} \quad \gamma = \gamma_i(\frac{T_\nu}{T_i})^2. \tag{2.3}$$

With this notation, the rate for the reaction $n + e^+ \to p + \bar{\nu}_e$, is given by

$$\begin{aligned}\lambda_a =&\frac{g_V^2(1+3\alpha^2)m_e^5\gamma_i T_\nu^2}{4\pi^3 T_i^2}\sum_{n_s=0}^{\infty}[2-\delta_{n_s0}]\int_{\sqrt{1+2\gamma n_s}+\kappa}^{\infty}\frac{d\epsilon(\epsilon-\kappa)}{\sqrt{(\epsilon-\kappa)^2-(1+2\gamma n_s)}}\\ &\times\frac{1}{(1+e^{\epsilon Z_e+\phi_e})}\frac{(\epsilon+q)^2 e^{(\epsilon+q)Z_\nu+\phi_\nu}}{(1+e^{(\epsilon+q)Z_\nu+\phi_\nu})},\end{aligned} \tag{2.4}$$

where $g_V^2(1+3\alpha^2)m_e^5/2\pi^3 \simeq 6.515\times 10^{-4}\text{sec}^{-1}$, $g_V = 1.4146\times 10^{-49}\text{erg cm}^3$, and $\alpha = g_A/g_V \simeq -1.262$.[12]

Similarly, for the reaction $n + \nu \to p + e^-$, the rate is

$$\begin{aligned}\lambda_b =&\frac{g_V^2(1+3\alpha^2)m_e^5\gamma_i T_\nu^2}{4\pi^3 T_i^2}\sum_{n_s=0}^{\infty}[2-\delta_{n0}]\int_{\sqrt{1+2\gamma n_s}+\kappa}^{\infty}\frac{d\epsilon(\epsilon-\kappa)}{\sqrt{(\epsilon-\kappa)^2-(1+2\gamma n_s)}}\\ &\times\frac{1}{(1+e^{\epsilon Z_e+\phi_e})}\frac{(\epsilon-q)^2 e^{\epsilon Z_e+\phi_e}}{(1+e^{(\epsilon-q)Z_\nu-\phi_\nu})}\\ &-\frac{g_V^2(1+3\alpha^2)m_e^5\gamma_i T_\nu^2}{4\pi^3 T_i^2}\sum_{n_s=0}^{n_{s\max}}[2-\delta_{n_s0}]\int_{\sqrt{1+2\gamma n_s}+\kappa}^{q}\frac{d\epsilon(\epsilon-\kappa)}{\sqrt{(\epsilon-\kappa)^2-(1+2\gamma n_s)}}\\ &\times\frac{1}{(1+e^{\epsilon Z_e+\phi_e})}\frac{(\epsilon-q)^2 e^{\epsilon Z_e+\phi_e}}{(1+e^{(\epsilon-q)Z_\nu-\phi_\nu})},\end{aligned} \tag{2.5}$$

and for the reaction $n \to p + e^- + \bar{\nu}_e$, we have

$$\lambda_c = \frac{g_V^2(1+3\alpha^2)m_e^5\gamma_i T_\nu^2}{4\pi^3 T_i^2} \sum_{n_s=0}^{n_{s\max}} [2-\delta_{n_s0}] \int_{\sqrt{1+2\gamma n_s}+\kappa}^{q} \frac{d\epsilon(\epsilon-\kappa)}{\sqrt{(\epsilon-\kappa)^2-(1+2\gamma n_s)}} \times \frac{1}{(1+e^{\epsilon Z_e+\phi_e})} \frac{(q-\epsilon)^2 e^{\epsilon Z_e+\phi_e}}{(1+e^{(q-\epsilon)Z_\nu+\phi_\nu})}. \tag{2.6}$$

The total weak reaction rates for the conversion of neutrons to protons is simply the sum of the above rates

$$\lambda_{n\to p} = \frac{g_V^2(1+3\alpha^2)m_e^5\gamma_i T_\nu^2}{4\pi^3 T_i^2} \sum_{n_s=0}^{\infty} [2-\delta_{n_s0}] \times \int_{\sqrt{1+2\gamma n_s}+\kappa}^{\infty} d\epsilon \frac{(\epsilon-\kappa)}{1+e^{\epsilon Z_e+\phi_e}} \times \frac{1}{[(\epsilon-\kappa)^2-(1+2\gamma n_s)]^{\frac{1}{2}}} [\frac{(\epsilon+q)^2 e^{(\epsilon+q)Z_\nu+\phi_\nu}}{1+e^{(q+\epsilon)Z_\nu+\phi_\nu}} + \frac{(\epsilon-q)^2 e^{\epsilon Z_e+\phi_e}}{1+e^{(\epsilon-q)Z_\nu-\phi_\nu}}]. \tag{2.7}$$

The parameters used above are defined as $\epsilon = \frac{E}{m_e}$, $q = \frac{m_n-m_p}{m_e}$, $Z_e = \frac{m_e}{T_e}$, $Z_\nu = \frac{m_e}{T_\nu}$, $\phi_e = \frac{\mu_e}{T_e}$, and $\phi_\nu = \frac{\mu_\nu}{T_\nu}$, where m_n and m_p are the rest masses of the neutron and proton respectively, T_e is the temperature of the electrons, $\mu_i (i = e,\, \nu)$ is the chemical potential of the electron or neutrino, and $\phi_i (i = e,\, \nu)$ is the degeneracy parameter.

For β-decay processes to occur in the presence of a magnetic field, the quantum number n_s has to satisfy

$$\sqrt{1+2\gamma n_s} + \kappa \le q, \quad \text{or} \quad n_s \le n_{s\max} = \text{Int}\left[\frac{(q-\kappa)^2-1}{2\gamma}\right], \tag{2.8}$$

where $n_{s\max}$ is the largest integer in $\frac{(q-\kappa)^2-1}{2\gamma}$. (Note that our expressions for the total rate (2.7) here is different from that given in Ref [4].)

The inverse total reaction rate of the conversion of protons to neutrons is computed as

$$\lambda_{p\to n} = e^{-qZ_e}\lambda_{n\to p}.$$

In order to better elucidate the effects of the field on the rates, we calculate analytically the variations of the reaction rates with respect to changes in the field, and we obtain

$$\lim_{\gamma\to 0} \frac{d\lambda_a}{d\gamma} \propto \sum_{n_s=0}^{\infty} [2-\delta_{n_s0}] \int_0^\infty dk f_+(\epsilon) u_+(\gamma,\epsilon) > 0, \tag{2.9}$$

$$\lim_{\gamma\to 0} \frac{d\lambda_b}{d\gamma} \propto \sum_{n_s=0}^{\infty} [2-\delta_{n_s0}] \int_0^\infty dk f_-(\epsilon) u_-(\gamma,\epsilon) - \sum_{n_s=0}^{n_{s\max}} [2-\delta_{n_s0}] \int_0^q dk f_-(\epsilon) u_-(\gamma,\epsilon) > 0, \tag{2.10}$$

$$\lim_{\gamma\to 0}\frac{d\lambda_c}{d\gamma}\propto\sum_{n_s=0}^{n_{\rm smax}}[2-\delta_{n_s0}]\int_0^q dk f_-(\epsilon)u_-(\gamma,\epsilon)>0, \tag{2.11}$$

and

$$\lim_{\gamma\to 0}\frac{d\lambda_{n\to p}}{d\gamma}\propto\sum_{n_s=0}^{\infty}[2-\delta_{n_s0}]\int_0^\infty dk[f_+(\epsilon)u_+(\gamma,\epsilon)+f_-(\epsilon)u_-(\gamma,\epsilon)]>0, \tag{2.12}$$

where

$$\epsilon=(k^2+1+2\gamma n_s)^{1/2}+\kappa, \qquad k=\frac{p_z^2}{m_e^2},$$

$$f_\pm(\epsilon)=\frac{(\epsilon\pm q)^2}{(1+e^{\pm(\epsilon Z_e+\phi_e)})(1+e^{-[(q\pm\epsilon)Z_\nu+\phi_\nu]})}.$$

$$u_\pm(\gamma,\epsilon)=1+\frac{2\gamma n_s}{\epsilon(\epsilon\pm q)}\mp\frac{\gamma n_s Z_e}{\epsilon}\frac{e^{\pm(\epsilon Z_e+\phi_e)}}{1+e^{\pm(\epsilon Z_e+\phi_e)}}\pm\frac{\gamma n_s Z_\nu}{\epsilon(1+e^{-[(q\pm\epsilon)Z_\nu+\phi_\nu]})}.$$

We also computed Eqs.(2.9 -2.12) numerically, for various of γ_i and T. Both calculations show that, independent of the temperature, the presence of a magnetic field does increase all the weak reaction rates, including the total neutron depletion rate. This result is consistent with the findings in our previous works and with Grasso and Rubinstein's recent calculations,[4] but inconsistent with Kernan's recent statement[5] that the rates of 2-2 processes decrease as the field increases. At very high temperatures $T \gg 2.5$ MeV, such effects are insignificant because the inverse reaction rates also increase with the field and are not much suppressed by the factor $\exp(-qZ_e)$. When the temperature drops to a point where the reactions $n+e^+\to p+\bar{\nu}$ and $n+\nu\to p+e^-$ begin to freeze out and the neutron β-decay process dominates, then the total rate increases with the magnetic field. However, if the primeval field is not strong enough to begin with, then as the universe expands, it becomes too weak to affect the reaction rates at low temperatures. Our numerical calculations reveal that for the magnetic field to have significant impact on the reaction rates, $\gamma_i \gtrsim 10^3$ or $B_i \gtrsim 4.4\times 10^{16}$ G at 1 MeV. As we discuss below, the effect due to the change in expansion rate is already significant for $\gamma_i \gtrsim 10$ and dominates over the change on the reaction rates.

2.2 Electron density phase space

In a magnetic field, the phase space and energy density of electrons are modified. The number density and energy density of electrons (n_e and ρ_e) over phase space as a function

of magnetic field strength are given by

$$n_e = 2\frac{m_e^3 \gamma_i T_\nu^2}{(2\pi)^2 T_i^2} \sum_{n_s=0}^{\infty} (2-\delta_{n_s 0}) \int_{\sqrt{1+2\gamma n_s}+\kappa}^{\infty} d\epsilon \frac{(\epsilon-\kappa)}{\sqrt{(\epsilon-\kappa)^2-(1+2\gamma n_s)}} \frac{1}{1+e^{\epsilon Z_e+\phi_e}} \tag{2.13}$$

and

$$\rho_e(B) = 2\frac{m_e^4 \gamma_i T_\nu^2}{(2\pi)^2 T_i^2} \sum_{n_s=0}^{\infty} (2-\delta_{n_s 0}) \int_{\sqrt{1+2\gamma n_s}+\kappa}^{\infty} d\epsilon \frac{\epsilon(\epsilon-\kappa)}{\sqrt{(\epsilon-\kappa)^2-(1+2\gamma n_s)}} \frac{1}{1+e^{\epsilon Z_e+\phi_e}}. \tag{2.14}$$

Correspondingly, the pressure of electrons is

$$P_e = 2\frac{m_e^4 \gamma_i T_\nu^2}{(2\pi)^2 T_i^2} \sum_{n_s=0}^{\infty} (2-\delta_{n_s 0}) \int_{\sqrt{1+2\gamma n_s}+\kappa}^{\infty} d\epsilon \frac{(\epsilon-\kappa)}{3\epsilon} \frac{\sqrt{(\epsilon-\kappa)^2-(1+2\gamma n_s)}}{1+e^{\epsilon Z_e+\phi_e}}. \tag{2.15}$$

These expressions will reduce to[10]

$$n_e = \frac{m_e^3}{\pi^2} \int_1^{\infty} d\epsilon \frac{\epsilon\sqrt{\epsilon^2-1}}{1+e^{\epsilon Z_e+\phi_e}}, \tag{2.16a}$$

$$\rho_e = \frac{m_e^4}{\pi^2} \int_1^{\infty} d\epsilon \frac{\epsilon^2\sqrt{\epsilon^2-1}}{1+e^{\epsilon Z_e+\phi_e}}, \tag{2.16b}$$

and

$$P_e = \frac{m_e^4}{3\pi^2} \int_1^{\infty} d\epsilon \frac{(\epsilon^2-1)^{3/2}}{1+e^{\epsilon Z_e+\phi_e}} \tag{2.16c}$$

if the magnetic field is absent.

The dependences of n_e, ρ_e, and P_e on the magnetic field can be seen analytically to be

$$\lim_{\gamma\to 0} \frac{dn_e}{d\gamma} \propto \sum_{n_s=0}^{\infty} (2-\delta_{n_s 0}) \int_0^{\infty} dk \frac{1}{1+e^{\epsilon Z_e+\phi_e}} [1 - \frac{\gamma n_s Z_e}{\epsilon(1+e^{-(\epsilon Z_e+\phi_e)})}] > 0, \tag{2.17}$$

$$\lim_{\gamma\to 0} \frac{d\rho_e}{d\gamma} \propto \sum_{n_s=0}^{\infty} (2-\delta_{n_s 0}) \int_0^{\infty} dk \frac{\epsilon}{(1+e^{\epsilon Z_e+\phi_e})} [1 + \frac{\gamma n_s}{\epsilon^2} - \frac{\gamma n_s Z_e}{\epsilon(1+e^{-(\epsilon Z_e+\phi_e)}))}] > 0, \tag{2.18}$$

and

$$\lim_{\gamma\to 0} \frac{dP_e}{d\gamma} \propto \sum_{n_s=0}^{\infty} (2-\delta_{n_s 0}) \int_0^{\infty} dk \frac{k^2}{\epsilon(1+e^{\epsilon Z_e+\phi_e})} [1 - \frac{\gamma n_s}{\epsilon^2} - \frac{\gamma n_s Z_e}{\epsilon(1+e^{-(\epsilon Z_e+\phi_e)})}] > 0. \tag{2.19}$$

where we assumed non-degenerate neutrinos ($\phi_\nu = 0$). These expressions indicate that, in the presence of magnetic fields, due to the large Landau excitation energy and the

decreased cross-sectional area of each Landau level, all of the electron thermodynamic quantities, such as n_e, ρ_e, and P_e, increase with increasing field strengths. This, in turn, causes a decrease in all of the weak interaction rates and changes the temperature-time relationship in BBN calculations. Furthermore, it results in an increase in the final neutron to proton ratio and the abundances of the light elements. Such an effect becomes significant for $\gamma_i \gtrsim 10^3$ and is sub-dominant to the effect discussed below.

2.3 Effects on the Expansion Rate

The expansion rate of our universe is given by

$$H \equiv \frac{1}{R}\frac{dR}{dt} = \sqrt{\frac{8\pi G}{3}\rho}, \tag{2.20}$$

where G is the gravitational constant and ρ is the total energy density. This can be expressed as $\rho = \rho_\gamma + \rho_e + \rho_\nu + \rho_b + \rho_B$, where $\rho_e = \rho_{e^-} + \rho_{e^+}$, $\rho_\nu = \rho_{\nu_e} + \rho_{\nu_\mu} + \rho_{\nu_\tau} + \rho_{\bar{\nu}_e} + \rho_{\bar{\nu}_\mu} + \rho_{\bar{\nu}_\tau}$, and the subscripts $\gamma,\ e,\ \nu_e,\ \nu_\mu,\ \nu_\tau,\ b,$ and B stand for photons, electrons, e-neutrinos, μ-neutrinos, τ-neutrinos, baryons, and magnetic fields.

The presence of magnetic fields alters the expansion rate by the added energy density of the magnetic field

$$\rho_B = \frac{B_c^2}{8\pi}\gamma_i^2\left(\frac{T_\nu}{T_i}\right)^4 \tag{2.21}$$

and the change in the electron energy density which we can write as

$$\rho_e \equiv \rho_e(B=0) + \delta\rho_e \tag{2.22}$$

During nucleosynthesis, e^+e^- annihilation transfers entropy to the photons but not to the decoupled neutrinos. The neutrino temperature then follows $T_\nu \propto R^{-1}$, i.e.,

$$\frac{dT_\nu}{dt} = -HT_\nu \tag{2.23}$$

while the photon temperature satisfies

$$\frac{dT_\gamma}{dt} = -3H\frac{\rho_e + \rho_\gamma + P_e + P_\gamma}{d\rho_e/dT_\gamma + d\rho_\gamma/dT_\gamma}\ . \tag{2.24}$$

These equations are solved simultaneously since H is a function of both T_ν and $T\gamma$, i.e.,

$$\rho(T_\gamma, T_\nu) = \rho_\nu(T_\nu) + \rho_e(T_\gamma, B(T_\nu)) + \rho_\gamma(T_\gamma) + \rho_B(T_\nu) + \rho_b(T_\gamma)\ . \tag{2.25}$$

If we define

$$\rho_0 \equiv \rho(B=0),\ \theta \equiv \frac{\delta\rho_e}{\rho_0},\ \chi \equiv \frac{\rho_B}{\rho_0}, \tag{2.26}$$

we can estimate the effect of θ and χ on the time-temperature relation away from e^+e^- annihilation:

$$T_\gamma \sim 10^9 K\xi\,(1+\theta+\chi)^{-1/4} t^{-1/2} \ . \tag{2.27}$$

$\xi = 4.7$ for three types of neutrinos. If there is no magnetic field, Eq.(2.27) reduces to the formula in standard BBN[12]

$$T_\gamma \approx 10^9 K\xi t^{-1/2} \, .$$

The modified time-temperature relationship (2.27) suggests that the contributions of the primordial magnetic field from both the field energy density and the electron energy density, accelerate the expansion rate of the universe and decrease the time scale over which BBN can occur. In particular, the neutrons will have less time to decay to protons than in the field-free case, which leads to an enhanced final $\frac{n}{p}$ ratio and ultimately elevates the abundance of ^4He.

Comparing the energy density from the electrons to that directly from magnetic fields, we find that for $\gamma_i < 10$, the contribution from electrons is somewhat greater than that from the field, but it is still too small to be interesting with respect to the total energy density in the field free case. When the B field is stronger, $\gamma_i > 10$, the contribution from the magnetic field exceeds that of the electron phase space and dominates the total energy density.

3. Limits on the field strength from BBN

We considered all three effects discussed above in our numerical calculations to set a limit on the field strength allowed by BBN considerations. As expected, our calculations reveal that the abundances of the light elements are manifestly affected by strong magnetic fields ($B_i \gtrsim 10^{15}$G). Although the three effects are important for fields in excess of B_c, the dominant process for setting an upper limit on the magnetic field during BBN is the change in the time-temperature relation discussed in §2.3 in agreement with the results of Refs.[4] and [5].

Our numerical calculations show that for an initial magnetic field $B(1\ \text{MeV}) \lesssim 10^{15}$G, the impact on the neutron to proton ratio from the magnetic field energy density, which decreases the neutron population, is more significant than the other effects. By using the observed abundance of ^{4}He, D, and ^{3}He, we find a constraint on the strength of a primordial magnetic field which is equivalent to an increase in the number of neutrino families. To explicitly calculate these effects, we set the neutron lifetime $\tau_n = 887$ s $\pm$

2 s,[13] the number of neutrino species $N_\nu = 3$, and compute the primordial abundances numerically. Similarly to the bound on N_ν, the constraint on the magnetic field energy relies on the lower limit to η and the upper limit to ^{4}He. We use the D + ^{3}He lower bound on the baryon-to-photon ratio ($\eta \geq 2.5 \times 10^{-10}$) and an upper limit to the ^{4}He abundance ($Y_P \leq 0.245$)[14] and find that $\gamma_i \leq 85$. This implies that the allowed magnetic field at the end of BBN ($T_\gamma = 0.01$ MeV) is less than about 2×10^9G which corresponds to a limit on the energy density of magnetic fields during BBN $\rho_B \leq 0.28\rho_\nu$.

4. Conclusion

In previous sections, we have provided a detailed analysis of the three major effects of a primordial magnetic field on the final abundances of the elements formed in big-bang nucleosynthesis. We have found that of the three major effects - (a) increased weak interaction rates; (b) enhanced electron densities in phase space; and (c) an increased expansion rate of the universe by the energy densities of magnetic field and electrons - the latter effect dominates over the modifications arising from the first two effects when $B_i \leq 10^{15}$G even when the electron magnetic moment is included. We have computed these effects numerically and obtained a revised upper limit on the allowed strength of a primordial magnetic field on scales smaller than l_d. Our results show that, in the framework of standard big bang nucleosynthesis, the maximum strength of a primordial magnetic field is such that $\rho_B \leq 0.28\rho_\nu$.

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